Pedestrian crowd models

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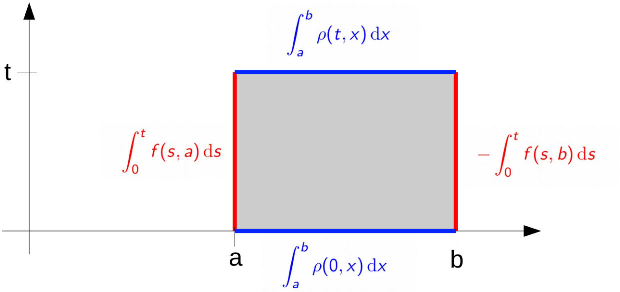
Outline

We want to model a moving crowd. The crowd is represented as a pedestrian density ρ between 0 and 1.

The agents flux is represented by the flux function f.



$$\int_a^b \rho(t,x)\,\mathrm{d}x = \int_a^b \rho(0,x)\,\mathrm{d}x + \int_0^t f(s,a)\,\mathrm{d}s - \int_0^t f(s,b)\,\mathrm{d}s$$

$$\int_a^b \int_0^t \partial_t \rho(s,x)\,\mathrm{d}s\,\mathrm{d}x = -\int_0^t \int_a^b \partial_x f(s,x)\,\mathrm{d}x\,\mathrm{d}s$$

The flux is equal to the density multiply by the speed of agents.

$$f(s,x) := \rho(s,x)v(s,x)$$

The velocity v is itself governed by the local density:

$$v(s,x) := v_{\max}(1-\rho)$$

We set $v_{\max} = 1$ and recover:

$$f(s,x) := f(\rho(s,x)) := \rho(s,x)(1-\rho(s,x))$$

• M. J. Lighthill and G. B. Whitham, On kinematic waves. ii. a theory of traffic flow on long crowded roads, (1955).

We end up with:

$$\int_a^b \int_0^t \partial_t \rho(s,x) + \partial_x f(\rho(s,x)) \,\mathrm{d}x \,\mathrm{d}s = 0$$

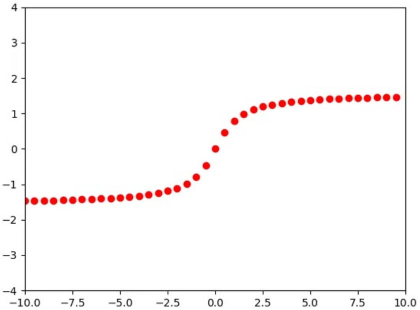
Short version, a scalar conservation law:

$$\rho_t + f(\rho)_x = 0$$

where $f(\rho) = \rho(1-\rho)$. What's known on this equation ?

- **Non-existence of continuous solutions**

We use a method of characteristics to propagate the initial datum:



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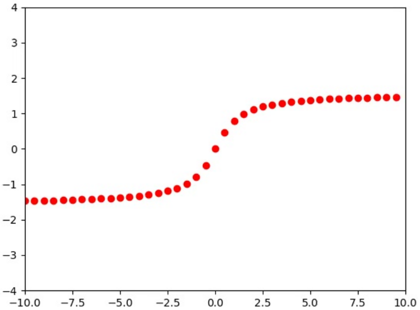


So we consider weak solutions :

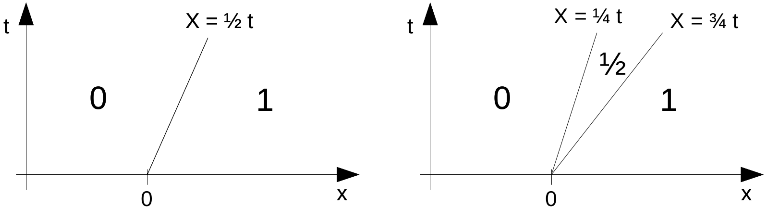
$$\forall \phi \in \mathcal{C}_c^\infty, \quad \iint_{(0,T)\times\mathbb{R}} \rho\phi_t + f(\rho)\phi_x \,\mathrm{d}t\,\mathrm{d}x = 0$$

- **Non-uniqueness of weak solutions**

Consider

$$\begin{cases} \rho_t + \left[\rho^2/2\right]_x = 0 \\ \rho(0,x) = \mathbb{1}_{(0,+\infty)} \end{cases} \tag{1}$$

Then the two density functions ρ described below are weak solutions:



Krushkov : entropy conditions
We say that $\rho \in L^\infty$ is an entropy solution to

$$\begin{cases} \rho_t + f(\rho)_x = 0 \\ \rho(0,\cdot) = \rho_0(\cdot) \in L^\infty \end{cases}$$

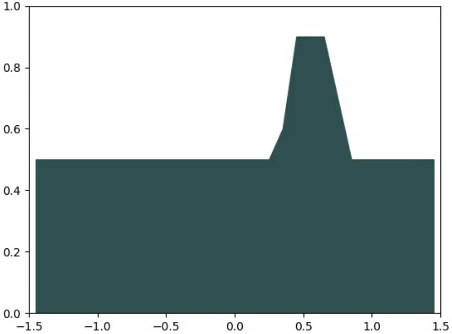
if

$$|\rho - k|_t + (\mathrm{sign}(\rho - k)\,(f(\rho) - f(k)))_x \leq 0 \text{ in the distributional sense.}$$

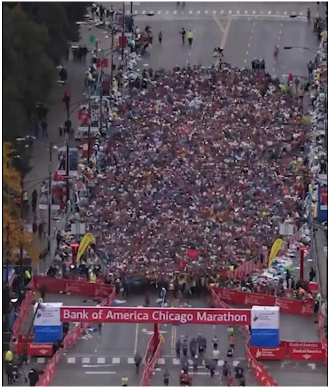
So $\forall k \in \mathbb{R},\ \forall \phi \in \mathcal{C}_c^\infty$

$$\iint_{(0,T)\times\mathbb{R}} |\rho - k|\phi_t + \mathrm{sign}(\rho - k)\,(f(\rho) - f(k))\,\phi_x \,\mathrm{d}t\,\mathrm{d}x + \int_{\mathbb{R}} |\rho_0 - k|\phi(0,x)\,\mathrm{d}x \geq 0$$

Interpretation of Kruskov entropy condition in the context of traffic: The admissible shocks correspond to the traffic jams.



How to model bottlenecks ?



Constrained evacuation :

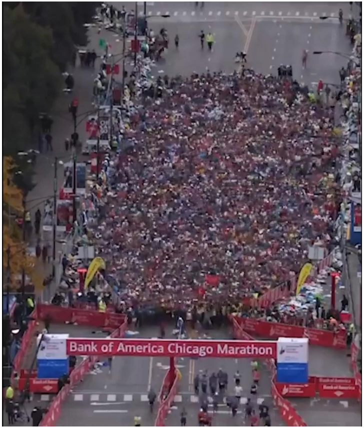
$$\begin{cases} \rho_t + f(\rho)_x = 0 \\ f(\rho(t, x = 0)) \leq Q \\ \rho(0, \cdot) = \rho_0(\cdot) \in L^\infty \end{cases} \tag{2}$$

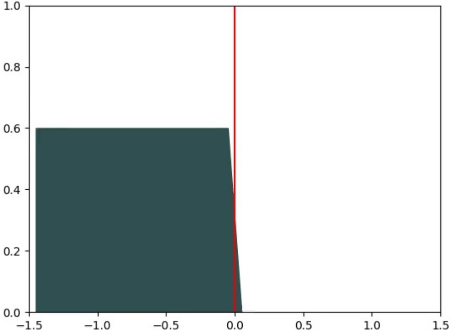
This is a **discontinuous-flux** scalar conservation law.

We say that $\rho \in L^\infty$ is a G-entropy solution if:

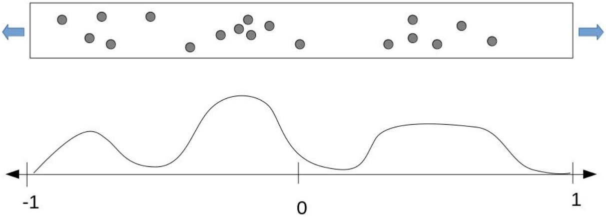
- ρ is a weak solution on the whole space.
- ρ satisfies the entropy inegualities on $(-\infty, 0)$ and $(0, +\infty)$.
- $(\rho(t, 0^-), \rho(t, 0^+)) \in G$ for almost every t.

R. M. Colombo and P. Goatin, *A well posed conservation law with a variable unilateral constraint*, J. Differ. Equ., 234 (2007), pp. 654–675.


Bank of America Chicago Marathon

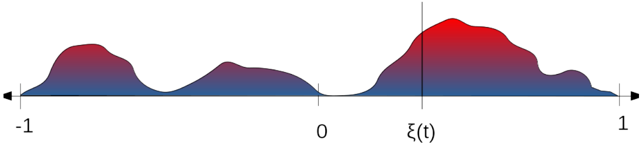

1.0
0.8
0.6
0.4
0.2
0.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5

How to model the psychology of crowds ?



A corridor with two doors located at $x = \pm 1$.

We consider a cost function c depending of the local density. We suppose each agent seeks to minimize its total cost towards the choosen exit.

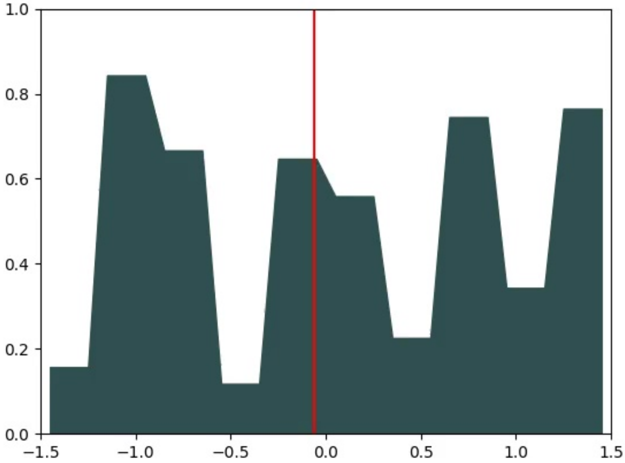


We want to solve:

$$\begin{cases} \rho_t + [\mathrm{sign}(x - \xi(t))\rho v(\rho)]_x = 0 \\ \int_{-1}^{\xi(t)} c(\rho(t,x))\,\mathrm{d}x = \int_{\xi(t)}^{1} c(\rho(t,x))\,\mathrm{d}x. \end{cases}$$

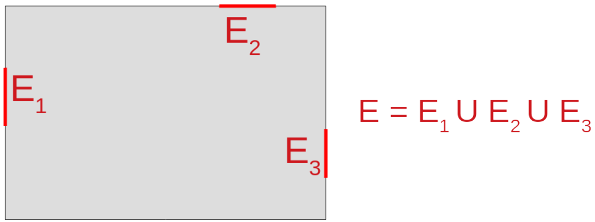
The curve ξ is called the turning curve. It is called the Hughes' model.

In the one-dimensional case, a fixed point argument proves the existence of (ρ, ξ) a solution.



Towards the 2D case

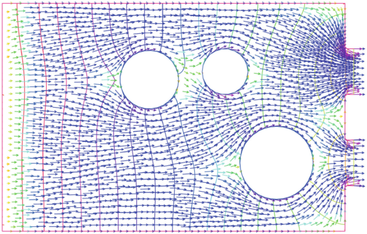
We want to study the crowd evacuation of an initial density ρ_0 in the room when, at $t = 0$, the agents want to exit the room minimizing their exit time.



Suppose $V(t, x)$ is a vector field corresponding to the choice of direction of an agent located in x at time t. Then the transport equation follows from LWR:

$$\rho_t + \mathrm{div}_x(V(t, x)\rho v(\rho)) = 0$$

How do we compute the trajectories of the pedestrians ?



2D: directions $\vec{V}$
Gokieli & al.'19

$$\begin{cases} |\nabla_x \phi| = c(\rho) \\ \phi(x \in E) = 0 \\ \nabla_x \phi \cdot n_D = 0 \text{ if } x \in \partial D \backslash E \end{cases} \tag{3}$$

where n_D is the normal unit vector to the boundary of the domain D and c is a given cost function depending on the local density.

work in progress...

Thank you.